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Review: Degenerate Series Representations of the q -deformed Algebra $soq'(r,s)$

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Groza, Valentyna A. [Groza, V. A.]

Degenerate series representations of the q -deformed algebra $\mathfrak{so}'_q(r, s)$. (English summary)

SIGMA Symmetry Integrability Geom. Methods Appl. **3** (2007), Paper 064, 12 pp.

This paper describes in detail certain representations of $\mathfrak{so}'_q(r, s)$, which is a real form of the complex q -deformed universal enveloping algebra $U'_q(\mathfrak{so}(n, \mathbb{C}))$ [A. M. Gavriliuk and A. U. Klimyk, *Lett. Math. Phys.* **21** (1991), no. 3, 215–220; [MR1102131 \(92k:17021\)](#)]. The latter algebra differs substantially in structure from the standard quantum algebra $U_q(\mathfrak{so}(n, \mathbb{C}))$ as defined by Drinfel'd and Jimbo; for instance it is not a Hopf algebra. However, as it is defined in terms of the generators $I_{k,k-1} = E_{k,k-1} - E_{k-1,k}$ for $\mathfrak{so}(n, \mathbb{C})$, it has the nice property that

$$U'_q(\mathfrak{so}(n-2, \mathbb{C})) \subset U'_q(\mathfrak{so}(n-1, \mathbb{C})) \subset U'_q(\mathfrak{so}(n, \mathbb{C})),$$

which in turn allows one to work with Gel'fand-Tsetlin type bases for finite-dimensional representations of $U'_q(\mathfrak{so}(n, \mathbb{C}))$, which have been studied in [N. Iorgov and A. U. Klimyk, *Int. J. Math. Math. Sci.* **2005**, no. 2, 225–262; [MR2143754 \(2006c:17021\)](#)].

In the paper under review, the author focuses on certain infinite-dimensional representations of $\mathfrak{so}'_q(r, s)$. These representations are described in terms of the finite-dimensional representations of $\mathfrak{so}'_q(r)$ and $\mathfrak{so}'_q(s)$, the compact real forms of $U'_q(\mathfrak{so}(r, \mathbb{C}))$ and $U'_q(\mathfrak{so}(s, \mathbb{C}))$, respectively. Explicit action formulas are given. An irreducibility criterion for the general construction is provided, along with a description of the irreducible constituents of the reducible ones coming from the same construction.

Reviewed by [Gizem Karaali](#)

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.